

Problem 1.10

[Difficulty: 4]

1.10 In a combustion process, gasoline particles are to be dropped in air at 200°F. The particles must drop at least 10 in. in 1 s. Find the diameter d of droplets required for this. (The drag on these particles is given by $F_D = \pi\mu Vd$, where V is the particle speed and μ is the air viscosity. To solve this problem, use *Excel's Goal Seek*.)

NOTE: Drag formula is in error: It should be:

$$F_D = 3 \cdot \pi \cdot V \cdot d$$

Given: Data on sphere and formula for drag.

Find: Diameter of gasoline droplets that take 1 second to fall 10 in.

Solution: Use given data and data in Appendices; integrate equation of motion by separating variables.

The data provided, or available in the Appendices, are:

$$\mu = 4.48 \times 10^{-7} \frac{\text{lb} \cdot \text{s}}{\text{ft}^2} \quad \rho_w = 1.94 \frac{\text{slug}}{\text{ft}^3} \quad \text{SG}_{\text{gas}} = 0.72 \quad \rho_{\text{gas}} = \text{SG}_{\text{gas}} \cdot \rho_w \quad \rho_{\text{gas}} = 1.40 \frac{\text{slug}}{\text{ft}^3}$$

Newton's 2nd law for the sphere (mass M) is (ignoring buoyancy effects)

$$M \cdot \frac{dV}{dt} = M \cdot g - 3 \cdot \pi \cdot \mu \cdot V \cdot d$$

so

$$\frac{dV}{g - \frac{3 \cdot \pi \cdot \mu \cdot d}{M} \cdot V} = dt$$

Integrating twice and using limits

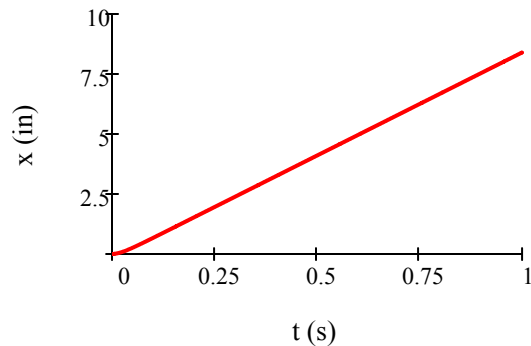
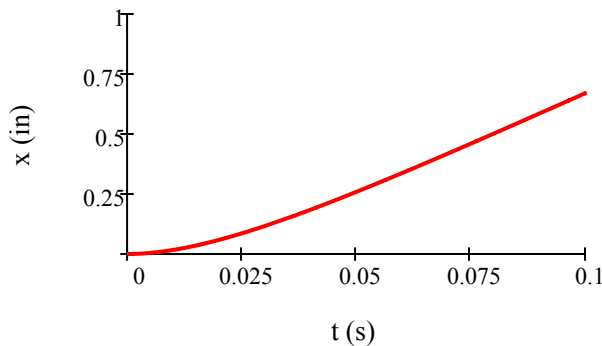
$$V(t) = \frac{M \cdot g}{3 \cdot \pi \cdot \mu \cdot d} \cdot \left(1 - e^{\frac{-3 \cdot \pi \cdot \mu \cdot d}{M} \cdot t} \right) \quad x(t) = \frac{M \cdot g}{3 \cdot \pi \cdot \mu \cdot d} \cdot \left[t + \frac{M}{3 \cdot \pi \cdot \mu \cdot d} \cdot \left(e^{\frac{-3 \cdot \pi \cdot \mu \cdot d}{M} \cdot t} - 1 \right) \right]$$

Replacing M with an expression involving diameter d

$$M = \rho_{\text{gas}} \cdot \frac{\pi \cdot d^3}{6} \quad x(t) = \frac{\rho_{\text{gas}} \cdot d^2 \cdot g}{18 \cdot \mu} \cdot \left[t + \frac{\rho_{\text{gas}} \cdot d^2}{18 \cdot \mu} \cdot \left(e^{\frac{-18 \cdot \mu}{\rho_{\text{gas}} \cdot d^2} \cdot t} - 1 \right) \right]$$

This equation must be solved for d so that $x(1 \text{ s}) = 10 \text{ in.}$ The answer can be obtained from manual iteration, or by using *Excel's Goal Seek*.

$$d = 4.30 \times 10^{-3} \text{ in.}$$



Note That the particle quickly reaches terminal speed, so that a simpler approximate solution would be to solve $Mg = 3\pi\mu Vd$ for d , with $V = 0.25 \text{ m/s}$ (allowing for the fact that M is a function of d)!